



A RESEARCH ON GEOMETRY PROBLEM SOLVING STRATEGIES USED BY ELEMENTARY MATHEMATICS TEACHER CANDIDATES

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Abstract

Geometry is one of the most important branches of mathematics education, because the aim of the geometry teaching is to provide students with the ability of critical thinking, problem solving and a better understanding of the other subjects in mathematics by making the students have a high level of geometric thinking skills (Şahin, O.,2008). Research of problem solving, which is located in the centre of education, in this extremely important branch of mathematics education is quite important for mathematics education in our country as in many countries. There is a lot of research on this issue in the literature. Elçin Emre(2008), has searched secondary school students' abilities of using problem solving strategies in her master's thesis and reached the conclusion that students are quite willing to use a strategy they have learned. Elçin Emre has also observed that students can use the strategies they have learned appropriately in their homework after the strategy teaching. On another research, İsrail (2003) has examined problem solving strategies used by 8th grade students in terms of level of success, gender, and socio-economic level variables and reached the conclusion that there is a significant relation between problem solving strategies and the level of success, socio-economic level and gender.

This study investigated elementary mathematics teacher candidates' problem solving strategies in geometry classes. The study was carried out with the participation of 20 students attending Buca Faculty of Education Elementary Mathematics Education program in 2012-2013 spring semester. In this study, semi-structured interview was used among qualitative research methods. In order to determine teacher candidates' problem-solving strategies, semi-structured interview form and "*the form of determining geometry problem solving strategies*" consisting of open-ended questions were developed as a means of data collection.

In the part of the research findings, there is data including problem solving strategies used by mathematics teacher candidates and the analysis of these strategies according to gender difference. According to the findings, it is determined that teacher candidates benefit from different problem solving strategies existing in the literature.

It is thought that investigation of problem solving, which has great importance in the field of mathematics as in many other areas, in geometry classes provides an important contribution to mathematics education by helping students develop their reasoning and problem solving skills, which is one of the aims of mathematics teaching, and providing these skills to be used later in life (Yılmaz, 2007).

Key Words: Geometry, problem solving, problem solving strategies.



INTRODUCTION

As the purpose of geometry teaching, which is one of the most important branches of mathematics education, is to provide students with the ability of critical thinking, problem solving, and a better understanding of the other subjects of mathematics by giving students a high level of geometrical thinking skills (Şahin, 2008), the place of geometry is quite large in our education system (Yılmaz & Turgut, 2007). The need for splitting a part of a plane correctly created geometry which is the knowledge of measuring objects and shapes and describing it with numbers. For this reason, the geometry course is quite related to people's daily lives. Therefore, geometry subjects usually attract people's attention and arouse interest in human (Fidan, 1986).

Problem is the case which evokes desire to solve in people, solution process does not exist but can be solved by using one's knowledge and experiences (Olkun & Toluk, 2004). According to Baykul (1999), problem is a work, in which an individual who is facing it feel the need for solving or want to solve it, s/he does not have a way how to solve it and s/he tries to solve it. We can usually describe problem solving as to produce simple applications of learned rules in order to formulize a new answer or develop a solution (Kılıç, 2003). However, problem solving should not be oversimplified as to answer a math question. Problem solving is a way of thinking, reasoning and using the things learned in all the math activities (Okur ve diğ., 2006). As understanding the information and establishing a relationship between information occurs in the problem solving process, problem solving is in the centre of education in resent years in our country as in many countries (Şahin, 2007). Problem solving is very important in mathematics classes as it is in many areas, because one of the aims of teaching mathematics is to develop students' thinking, problem solving skills and make these skills be used later in life (Yılmaz, 2007). The developments in the field of mathematics show that students must acquire the ability of problem solving. The students who acquire the ability and habit of problem solving in school years take part as individuals who can take care of the problems in the community life in the future (Şahin, 2007).

There are many factors that affect problem solving but one of the most important of these factors is to choose and use the appropriate strategy. Therefore, the strategy for each problem is different. The appropriate strategy makes the problem solvers think about the meanings of both problem sentence and the mathematical equation. Problem solving is the work of establishing a correlation between the things given and requested. Establishing this correlation correctly happens with the help of strategies. According to the researchers, strategies are containable and conscious activities which carry out cognitive objectives (Pressley, 1995). As many strategies can be used for a type of problem, a strategy can also be used for many problems (Şahin, 2007). Some of the strategies that are often used for the solutions of geometry problems and their definitions are like that;

Making a Drawing: What is meant by the word drawing here is all the drawings that help the correlation and the data given in the problem become visible. These can be simple lines, geometrical shapes, dots, etc.(Arslan, 2002).

Intelligent Guessing and Testing Strategy: With this strategy, the answer of the problem is guessed while solving a problem and whether the guess is correct or not is tested. If it is correct, the problem is solved. If it is incorrect, new guesses are made. This process goes on until the correct answer is found(Altun, 2002).

Simplifying the problem: The problem is divided into sub-problems when encountered with wide complex problems in this strategy. Each sub-problem is any problem that simplifies the solution of the original problem. The successive simplifying process goes on until all the sub-problems are solved easily. Then these separated parts are recombined for the solution of the original problem(Dhillon, 1998).



Using Known Information: When solving a problem, we sometimes use the formula, correlation or relationship we know beforehand.

Brainstorming: Brainstorming is a good strategy for raising the quality and the number of the solution. First the problem is defined, and then all the possible solutions are put forward uncritically. Then by making criticism, the most applicable and practical solution is estimated and the best one is chosen (Dhillon, 1998).

The issue of problem solving, which is so important as to take place in the center of mathematics education, is fairly investigated by educators. When we look at the literature, Elçin Emre (2008) has investigated secondary school students' skills of using problem solving strategies in her master's thesis and reached the conclusion that students are quite willing to use a strategy they have learned. Elçin Emre has also observed that students can use the strategies they have learned appropriately in their homework after the strategy teaching. In his study, Israel (2003) has examined problem solving strategies used by 8th grade students in terms of level of success, gender, and socio-economic level variables and reached the conclusion that there is a significant relation between problem solving strategies and the level of success, socio-economic level and gender. Yazgan (2007) has conducted research on intelligent guessing and testing, making a drawing, finding a correlation, simplifying the problem, making a systematic list and working backwards strategies with 4th and 5th grade students who participated in the survey. As a result of this study, it is seen that students can use guessing and testing, making a drawing, making a systematic list and working backwards strategies without difficulty. The ones students had difficulty in while practicing are determined as finding a correlation and simplifying the problem strategies.

In the study conducted by Altun and the others (2007), the thoughts of the students in teacher training programs about problem solving strategies were examined besides the effects of the training whose topic is problem solving strategies on the problem solving success. The teaching has been effective for the teaching of all the strategies except for writing equations and reasoning and led to an increase in problem solving success. It is determined that the success of problem solving can be explained by three factors and it is concluded that the strategies that are strong in pointing the problem solving success are as follows respectively; finding a correlation, working backwards, simplifying the problem, making a systematic list, reasoning and drawing a diagram. All the students indicated that teaching of the strategies that are the subject of the study must be included in teacher training.

Although there are quite a lot of source on problem solving in the literature, there are limited resources about elementary school mathematics teacher candidates' problem solving skills and strategies. Whereas, the investigation of elementary school mathematics teacher candidates' problem solving skills and strategies, which will be one of the most important factors in the development of problem solving skills that is in the centre of the objectives of elementary school mathematics classes, is very important for mathematics education. Also, the study of the geometry problems, which have always had an important place in our lives and been relevant to our daily lives, will be quite important for mathematics education. For this reason, the investigation of elementary school mathematics teacher candidates' geometry problem solving strategies will provide an important contribution to mathematics education.

Aim

In this study, the geometry problem solving strategies used by elementary mathematics teacher candidates were tried to determine.

METHOD

In this study, descriptive research technique was used to examine research problem. This method is used for the research that try to describe and explain the cases' forms, actions and changes and their similarities and differences with other cases (Gall, Borg, & Gall, 1996). In this study, semi-structured interview was used among qualitative research methods and open ended questions were asked to determine the students' problem solving strategies.

The Working Group

The working group consists of 20 teacher candidate students, 10 girls and 10 boys, who were the first year students at Buca Faculty of Education Elementary Mathematics Education program in 2012-2013 spring semester.

Data Collection Tool

The data of the study was collected by using “the form of determining geometry problem solving strategies” consisting of open-ended questions and semi-structured interview form to determine teacher candidates’ problem-solving strategies. While the data collection tools were prepared, curriculum, textbooks, NTCM criteria were taken into consideration and expert opinions were obtained.

FINDINGS AND COMMENTS

The percentages of the findings on the strategies used by the teacher candidates in accordance with the interviews with teacher candidates and the scale applied to teacher candidates were shown in Table 1.

Table 1: Percentage of elementary mathematics teacher candidates' use of strategies

Strategies	Number of People Using The Strategy	Percentage
Making a Drawing	20	100 %
Intelligent Guessing and Testing	7	35 %
Brainstorming	8	40 %
Using Known Information	20	100 %
Simplifying the problem	13	65 %

According to the data obtained, all the teacher candidates participating in the study used making a drawing strategy while solving a geometry problem. The question “*In some geometry questions, do you try to improve the question visually or in order to see the question visually do you change the question into a visual expression by using the data in cases where the data is verbal?*” was asked in the interview to identify whether the teacher candidates use making a drawing strategy or not and some of the answer they give are like that;

S.1- *Yes. Because when I see the question visually, I solve it easier and I find the solution more quickly.*

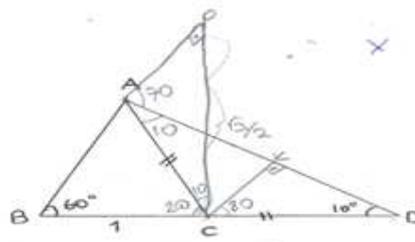
S.2- *Yes. I solve the question by changing it into a visual. Because geometry questions are more about seeing than memorizing.*

S.3- *Yes. I think about where to draw the lines. I change a verbal question into visual and then I solve it, I have difficulty in reaching the solution if I cannot see.*

S.4- *Yes. I like drawing and solving the questions is more attractive to me with the shapes I draw, it makes me relax.*

Besides, some of the problem solutions that are in the form of determining the geometry problem solving strategies and teacher candidates use making a drawing strategy, are like that;

S.5



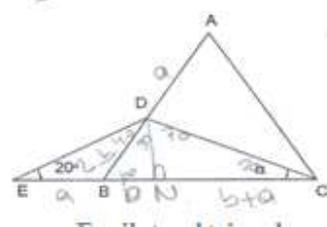
$m(\widehat{ABD}) = 30^\circ$,
 $m(\widehat{ADB}) = 90^\circ$,
 If,
 $|AC| = |CD|$ and
 $|BC| = 1 \text{ cm}$ $|AD| = ?$

$- 30^\circ - 60^\circ - 90^\circ$
 $|BC| = 1 \Rightarrow |PC| = \frac{1}{2}$

$\triangle PAC \cong \triangle KCD$
 $|KD| = \frac{1}{2}$
 $|AD| = \frac{1}{2}$

$\triangle PAC \cong \triangle KCB$
CDA triangle will be isoscles triangle
 so, $|AK| = |KD|$ $|AD| = \sqrt{3}$

S.9.



ABC Equilateral triangle
 $|EB| = |AD|$
 $m(\widehat{E}) = 20^\circ$
 $m(\widehat{ECD}) = ?$

If; $|AB| = 2b + a$, $|AD| = a$ And $|DB| = 2b$
 From 30-60-90 triangle $|BN| = b$
 So; $|EN| = |NC|$ and (DEC) triangle will be isoscles triangle.
 So; $\alpha = 20^\circ$

35 % of the teacher candidates involved in the research used guessing and testing strategy while solving geometry problems. The question "In some geometry questions, thinking that the answer of the question should be in a certain interval, do you check whether it is correct or not by putting the values in that interval into the solution without making an operation?" was asked in the interview to identify whether the teacher candidates use guessing and testing strategy or not and some of the answers given are like that;

S.3- My aim is to solve the question. I cannot find the solution of the question by guessing.

S.4- I never do. I try to solve the question directly in order not to waste time. Guessing and testing that value takes too much time.

S.6- Yes. First, thinking an interval, I find the value interval for myself, I try to reach the result by checking the values in that interval.

S.7- Yes, I sometimes do. For example; thinking that the answer is between 20 and 25, I try each and every value in that interval.

40 % of the teacher candidates involved in the research used brainstorming strategy while solving geometry problems. The question "While solving a geometry problem, thinking of several ways to solve the problem, do

you try to reach the solution by choosing the most applicable and practical one among them?" was asked in the interview to identify whether the teacher candidates use brainstorming strategy or not and some of the answers given are like that;

S.1- Yes. I generally find several ways for solution but I choose the most practical way. I choose the way that the least likely to make mistakes for me.

S.2- No. I choose the first way that comes to my mind when I see the question, I am not much interested in whether the way of solution is long or short..

S.6- No. I apply the first one that comes to my mind. For me, it is the most practical way. I can reach the result easily in that way, so I do not think of several ways for solution. Only if I cannot reach the solution in that way, I think of another way.

S.7- Yes. I think of a lot of ways for solution and then I choose the one which will give me the short and certain solution.

All the teacher candidates participating in the study used using known information strategy while solving a geometry problem. The question "To reach the solutions of geometry questions, do you benefit from the formulas, correlations or relationships you have already known?" was asked in the interview to identify whether the teacher candidates use using known information strategy or not and some of the answers they give are like that;

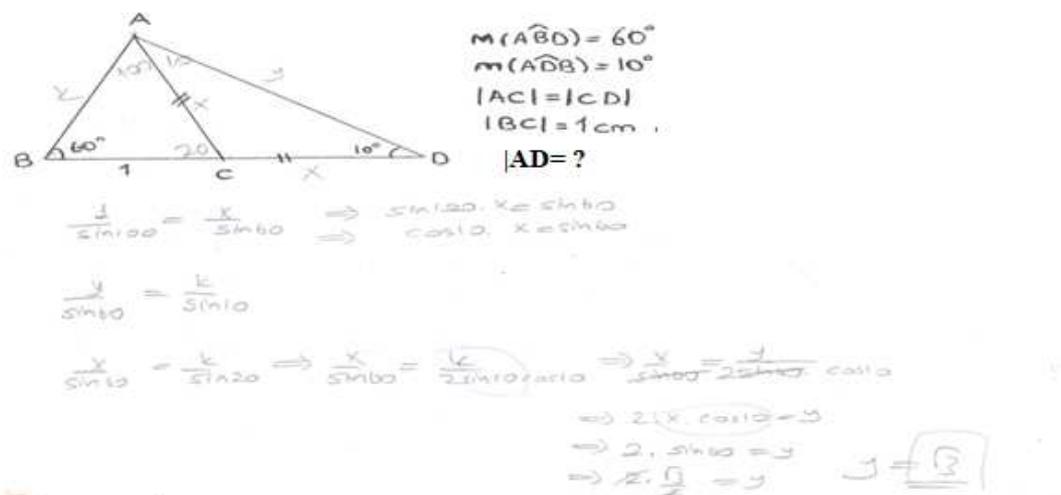
S.2- Yes. If I encounter with very complex operations while solving the question, I use the formulas and correlations I previously know and I reach the solution more easily. Rather than making a long operation, I use the formula if I know it to be more practical.

S. 6- Yes, I do. I solve the problem and reach the solution more easily by using a formula.

S.7- Yes. I generally use the formulas because formulas sound more accurate to me.

Also, some of the problem solutions that are in the form of determining the geometry problem solving strategies and teacher candidates use using known information strategy, are like that;

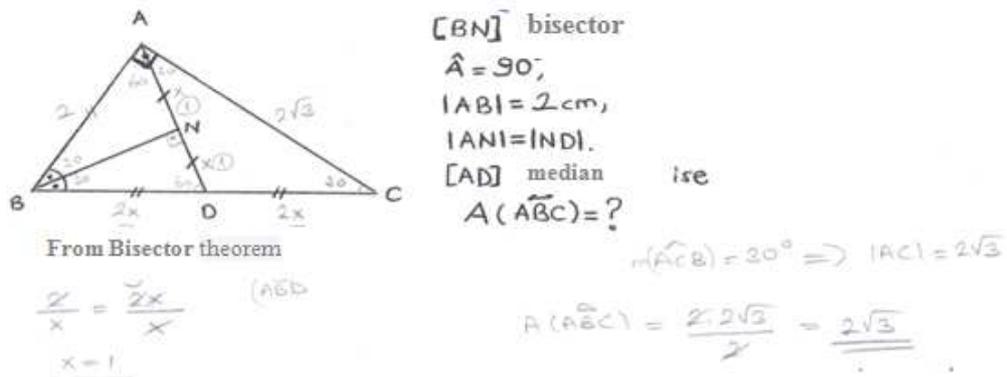
S.10



$m(\widehat{ABD}) = 60^\circ$
 $m(\widehat{ADB}) = 10^\circ$
 $|AC| = |CD|$
 $|BC| = 1 \text{ cm}$
 $|AD| = ?$

$\frac{1}{\sin 100} = \frac{x}{\sin 60} \Rightarrow \sin 120 \cdot x = \sin 10$
 $\Rightarrow \cos 10 \cdot x = \sin 10$
 $\frac{x}{\sin 10} = \frac{1}{\sin 10}$
 $\frac{x}{\sin 10} = \frac{1}{\sin 20} \Rightarrow \frac{x}{\sin 10} = \frac{1}{2 \sin 10 \cos 10} \Rightarrow \frac{x}{\sin 10} = \frac{1}{2 \sin 10 \cos 10}$
 $\Rightarrow 2(x \cdot \cos 10) = 1$
 $\Rightarrow 2 \cdot \sin 60 = 1$
 $\Rightarrow 2 \cdot \frac{\sqrt{3}}{2} = 1$
 $1 = 1$

S.12



[BN] bisector
 $\hat{A} = 90^\circ$;
 $|AB| = 2 \text{ cm}$;
 $|AN| = |ND|$.
 [AD] median ise
 $A(\widehat{ABC}) = ?$

From Bisector theorem

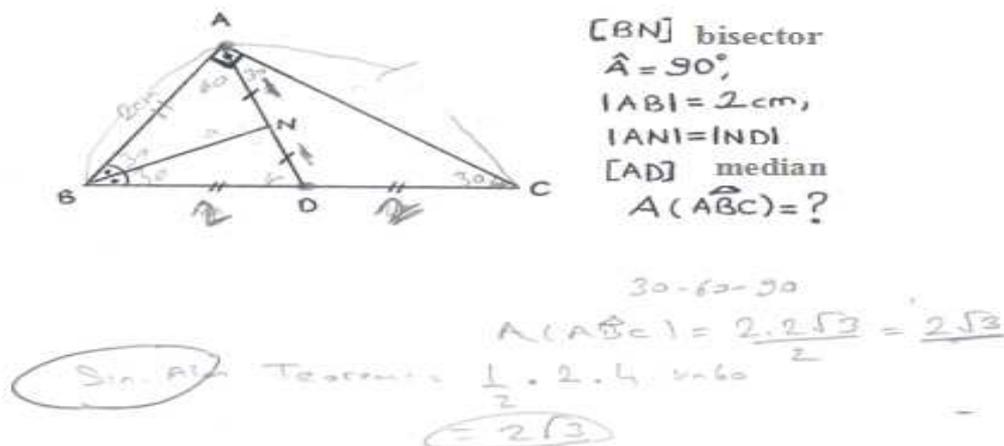
$$\frac{2}{x} = \frac{2x}{x} \quad (\widehat{AB})$$

$$x = 1$$

$m(\widehat{ACB}) = 30^\circ \Rightarrow |AC| = 2\sqrt{3}$

$$A(\widehat{ABC}) = \frac{2 \cdot 2\sqrt{3}}{2} = \underline{2\sqrt{3}}$$

S.19



[BN] bisector
 $\hat{A} = 90^\circ$;
 $|AB| = 2 \text{ cm}$;
 $|AN| = |ND|$.
 [AD] median
 $A(\widehat{ABC}) = ?$

30-60-90

$$A(\widehat{ABC}) = \frac{2 \cdot 2\sqrt{3}}{2} = \underline{2\sqrt{3}}$$

Sin-Ak Teoremi 1 * 2 * 4 = 8

$$= \underline{2\sqrt{3}}$$

65 % of the teacher candidates involved in the research used simplifying the problem strategy while solving geometry problems. The question “Do you try to solve a complex geometry problem by breaking it into simpler parts?” was asked in the interview to identify whether the teacher candidates use simplifying the problem strategy or not and some of the answers they give are like that;

S. 2- Yes. I break it into small parts if the question is too complex and I try to reach the whole. For example, while finding the area of octagon, I draw a square out of it and I find the whole area based on the small squares.

S.5- Yes. I generally solve the complex questions by breaking them into parts. I do like that with polygons or complex triangle questions.

S.6- Yes. For example, I can solve a question with hexagon by breaking it into triangles.

S.8- Yes. Going from part to whole is more logical. For example, in order to find the area of the trapezoid, it is easier to find the area of the two triangles, drawing a diagonal, than using a formula.

The percentages of the findings on the strategies used by the teacher candidates according to the gender are given in Table 2.

Table 2: The percentages of elementary mathematics teacher candidates' use of strategies according to the gender

Strategies	Female	Male
Drawing a Shape	100 %	100 %
Guessing and Testing	30 %	40 %
Brainstorming	20 %	60 %
Using Known Information	100 %	100 %
Simplifying the problem	70 %	60 %

According to the percentages of strategy use, it is observed that all the male and female teacher candidates use making a drawing and using known information strategy. While 40 % of the male teacher candidates use guessing and testing strategy while solving geometry problems, 30 % of the female teacher candidates use this strategy. While 60 % of the male teacher candidates use brainstorming, 20 % of the female teacher candidates benefit from this strategy. It is determined that male teacher candidates use guessing and testing strategy and brainstorming strategy more than female teacher candidates. Besides, while 70 % of the female teacher candidates use simplifying the problem strategy while solving geometry problems, 60 % of the male teacher candidates use this strategy. It is determined that female teacher candidates use simplifying the problem strategy more than male teacher candidates.

CONCLUSIONS, DISCUSSION AND SUGGESTIONS

According to the findings of the study, it is determined that all the elementary mathematics teacher candidates participated in the research use making a drawing and using known information strategies and 65 % of them use simplifying the problem strategy while solving geometry problems. It is observed that while 20 % of the female teacher candidates participated in the research benefit from brainstorming strategy while solving geometry problem, 60 % of the male teacher candidates benefit from this strategy. It is seen that 40 % of teacher candidates use brainstorming strategy overall. It is also determined that 35 % of the elementary mathematics teacher candidates participated in the study use guessing and testing strategy while solving geometry problems. In accordance with the results obtained in this study, recommendations below are presented.

- Different studies may also be conducted in order to determine the different strategies (Solving a Simpler Analogous Problem, Cognitive Research, etc..) used by elementary mathematics teacher candidates in geometry problems.
- Considering the importance of problem solving, it will be useful to give training to teacher candidates because problem solving is an integral part of the mathematics learning process and it is necessary to mention in related classes that problem solving should integrate with the overall mathematics learning process (Kayan & Çakıroğlu, 2008).
- Teachers should be a model to students by using different problem solving strategies in geometry classes and they should encourage students to use different strategies, so it is thought to be useful to give training to teachers and teacher candidates on teaching problem solving strategies.
- Research of secondary school students' geometry problem solving strategies can also be suggested.
- The relation between problem solving strategies used by the teacher candidates and students' problem solving strategies can be studied.



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